Dynamics of an Aircraft in Wind Shear of Arbitrary Direction

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This paper investigates dynamic characteristics of an aircraft in wind shear of an arbitrary direction. The steady-state solution allowing for the wind shear is defined and the linearized system about this solution is investigated. Wind shear accounts for coupling the longitudinal and the lateral-directional motions and for an additional mode as well. The thrust required for the steady-state motion, the characteristics of the modes, and the state variables they affect are decisively influenced by the wind shear intensity and direction and the vertical velocity of the aircraft. The new mode is either an aperiodic divergent one corresponding to the consideration of a specific azimuth angle as an additional state variable, or an oscillatory mode obtained by the coupling of the former with the spiral mode.

Nomenclature

a_W^E = acceleration of the aircraft center of mass with respect to F_E, written as components in F_W
C = aerodynamic side force
D = drag
F_B = body-fixed reference frame (body axes $Oxyz$)
F_E = Earth-fixed reference frame (Earth axes $O_E x_E y_E z_E$)
F_I = inertial reference frame
F_V = vehicle-carried vertical frame
F_W = air-trajectory reference frame (wind axes $Ox_W y_W z_W$)
g = gravity acceleration
I_x, I_y, I_z, I_{zx} = mass moments and product of inertia with respect to body axes
I_1 $= I_z I_x - I_{zx}^2$
I_2 $= I_{zx}(I_x - I_y + I_z)$
I_3 $= I_z(I_y - I_z) - I_{zx}^2$
I_4 $= I_x(I_x - I_y) + I_{zx}^2$
L = rolling moment
L = lift
L_{BV} = transformation matrix, relating vectors in F_V (or F_E) to those in F_B (see Appendix)
L_{WB} = transformation matrix, relating vectors in F_B to those in F_W (see Appendix)
L_{WV} = transformation matrix, relating vectors in F_V (or F_E) to those in F_W (see Appendix)
M = pitching moment
m = aircraft mass
N = yawing moment
p,q,r = scalar components of ω^A in body axes F_B
p^A, q^A, r^A = scalar components of ω in body axes F_B
T = thrust
t = time
u,v,w = scalar components of V in body axes F_B
u^W, v^W, w^W = scalar components of W^E
V = magnitude of V
V = velocity vector of the aircraft c.g. relative to the air
V^E = velocity vector of the aircraft c.g. relative to the Earth
W = magnitude of W^E
W^E = windspeed relative to the Earth at the aircraft c.g.
x,y,z = give position of c.g. relative to the reference frame
z_T = thrust vector arm
α = angle of attack
α_T = angle between thrust vector line and the x body axis
β = angle of sideslip
δ_a = aileron deflection angle
δ_e = elevator deflection angle
δ_r = rudder deflection angle
ϕ, θ, ψ = Euler angles [in Eq. (7)]
ψ = angle between x body axis and vector W^E, in a horizontal plane
ω = angular velocity of F_B relative to F_E
ω^A = angular velocity of the aircraft relative to the air
ω^W = angular velocity of F_W relative to F_E

Subscripts

E, W, B = reference frame in which the components of the vector are taken
t, x, y, z = partial derivative due to the respective quantity
ij = element of the matrix in the ith ($i,j = 1,2,3$) row and jth column

Superscripts

T = transpose
$(\dot{\ })$ = total time derivative

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Introduction

THE influence of wind shear on flight safety near the ground has been a constant concern of many authors. Some of them use numerical response simulation or energy methods for predicting the trajectory or the required thrust.[1,2] Others assume the aircraft initially trimmed for flight in the presence of a uniform wind, and compute the response of the linearized system about this reference equilibrium flight condition, with linear wind shear as a perturbation.[3] Etkin[4] develops a method for the estimation of longitudinal dynamic qualities in horizontal flight parallel to a wind of constant $\partial W/\partial z_E$, in which an additional aerodynamic derivative accounts for the effect of vertical wind gradient. This derivative is $d/dz_E = \ldots + (\partial/\partial V)\ (\partial V/\partial z_E)$, where $\partial V/\partial z_E = \partial W/\partial z_E$. As for the lateral-directional stability, he suggests estimating the stability derivatives with an allowance for the airspeed distribution on the airplane.

In this paper, a steady-state solution for flight in wind shear is defined and some of its flying qualities investigated. A numerical example is computed. The result is then compared with that obtained by using Etkin's z-derivative method.

As is known,[5] the atmospheric motion is characterized by the velocity due to the solid-body rotation of the air, and by that due to the axial and shear rates of strain superimposed on the velocity at the origin. Notwithstanding that there is no a priori knowledge concerning the relative importance of the rotation and the shear strain, only the velocity field associated with solid-body motion is considered in this paper as an illustration of the method put forth. The lack of sufficient information by the author concerning the aerodynamic data due to the strain caused the omission of considering their effect, although this would imply no difficulty as far as both the theory or its application are concerned.

Equations of Motion

Most of the reference frames and symbols are those used by Etkin in Ref. 4. Motion relative to F_I is considered. Forces are taken as components in F_W and moments as components in F_B.

The groundspeed is expressed as

$$V^E = V + W^E \tag{1}$$

and the acceleration of the aircraft c.g. in F_W is

$$a_W^E = L_{WV}\dot{V}_E^E = \dot{V}_W + \omega_W^W x V_W + L_{WV}\dot{W}_E^E \tag{2}$$

where

$$\omega_W^W = L_{WB}\begin{vmatrix} p \\ q-\dot{\alpha} \\ r \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \dot{\beta} \end{vmatrix} \tag{3}$$

Aerodynamic forces and moments are functions of height, airspeed, the aircraft's angular speed with respect to the air (ω^A), and controls.

The vector $\omega^A = |p^A, q^A, r^A|^T = \omega - \frac{1}{2}$ **rot** W^E and written as components in F_B:

$$\omega_B^A = \omega - \tfrac{1}{2} L_{BV}\ \mathbf{rot}\ W_E^E \tag{4}$$

where

$$\mathbf{rot}\ W_E^E = \begin{vmatrix} w_{Ey}^W - v_{Ez}^W \\ u_{Ez}^W - w_{Ex}^W \\ v_{Ex}^W - u_{Ey}^W \end{vmatrix} \tag{5}$$

The scalar components of this vector are

$$p^A = p - \tfrac{1}{2}(L_{BV})_{11}(w_{Ey}^W - v_{Ez}^W) - \tfrac{1}{2}(L_{BV})_{12}(u_{Ez}^W - w_{Ex}^W) - \tfrac{1}{2}(L_{BV})_{13}(v_{Ex}^W - u_{Ey}^W) \tag{6a}$$

$$q^A = q - \tfrac{1}{2}(L_{BV})_{21}(w_{Ey}^W - v_{Ez}^W) - \tfrac{1}{2}(L_{BV})_{22}(u_{Ez}^W - w_{Ex}^W) - \tfrac{1}{2}(L_{BV})_{23}(v_{Ex}^W - u_{Ey}^W) \tag{6b}$$

$$r^A = r - \tfrac{1}{2}(L_{BV})_{31}(w_{Ey}^W - v_{Ez}^W) - \tfrac{1}{2}(L_{BV})_{32}(u_{Ez}^W - w_{Ex}^W) - \tfrac{1}{2}(L_{BV})_{33}(v_{Ex}^W - u_{Ey}^W) \tag{6c}$$

If the kinematical relationships that give the Euler angle rates be added, the following equations are obtained

$$\begin{aligned}
\dot{V} &= -D/m - (L_{WV})_{11}\dot{u}_E^W - (L_{WV})_{12}\dot{v}_E^W - (L_{WV})_{13}(\dot{w}_E^W - g) + T/m\cos(\alpha_T + \alpha)\cos\beta \\
\dot{\beta} &= -C/(mV) + p\sin\alpha - r\cos\alpha - (L_{WV})_{21}/V\dot{u}_E^W - (L_{WV})_{22}/V\dot{v}_E^W - (L_{WV})_{23}/V(\dot{w}_E^W - g) - T/(mV)\cos(\alpha_T + \alpha)\sin\beta \\
\dot{\alpha} &= -L/(mV\cos\beta) + q - (p\cos\alpha + r\sin\alpha)\tan\beta - (L_{WV})_{31}/(V\cos\beta)\dot{u}_E^W \\
&\quad - (L_{WV})_{32}/(V\cos\beta)\dot{v}_E^W - (L_{WV})_{33}/(V\cos\beta)(\dot{w}_E^W - g) - T/(mV\cos\beta)\sin(\alpha_T + \alpha) \\
\dot{p} &= I_z/I_1 \mathrm{L} + I_{zx}/I_1 \mathrm{N} + I_2/I_1 pq + I_3/I_1 qr \\
\dot{q} &= \mathrm{M}/I_y + I_{zx}/I_y(r^2 - p^2) + (I_z - I_x)/I_y pr - Tz_T/I_y \\
\dot{r} &= I_{zx}/I_1 \mathrm{L} + I_x/I_1 \mathrm{N} + I_4/I_1 pq - I_2/I_1 qr \\
\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\
\dot{\theta} &= q\cos\phi - r\sin\phi \\
\dot{\psi} &= (q\sin\phi + r\cos\phi)\cos\theta
\end{aligned} \tag{7}$$

$$\dot{x}_E = (L_{WV})_{11}V + u_E^W \tag{8a}$$

$$\dot{y}_E = (L_{WV})_{12}V + v_E^W \tag{8b}$$

$$\dot{z}_E = (L_{WV})_{13}V + w_E^W \tag{8c}$$

$$\dot{u}_E^W = u_{Et}^W + (V_E^E)^T \mathbf{grad} u_E^W = u_{Et}^W + u_{Ex}^W \dot{x}_E + u_{Ey}^W \dot{y}_E + u_{Ez}^W \dot{z}_E \tag{9a}$$

$$\dot{v}_E^W = v_{Et}^W + (V_E^E)^T \mathbf{grad} v_E^W \tag{9b}$$

$$\dot{w}_E^W = w_{Et}^W + (V_E^E)^T \mathbf{grad} w_E^W \tag{9c}$$

The motion equations of the controls and the equations that define the aerodynamic forces and moments may also be added.

Windspeed variation in the atmosphere has been studied in Refs. 2, 6, and 7. In most cases, it may safely be assumed that $\mathbf{rot}\,W_E^E = \mathrm{const}$ and $\dot{W}_E^E = \mathrm{constant}$ along the reference equilibrium flight path in some region of the boundary layer. It can also be assumed that this region is sufficiently large to allow considering steady-state solutions of Eq. (7).

Specification of the Aerodynamic Forces and the Wind Field

As an example a light trainer airplane in landing and takeoff configuration is now considered. The gross weights are 3,000 and 4,000 daN, respectively. The aerodynamic coefficients for Mach = 0.2 have the following form

$$c_D = c_{Do} + c_{D\alpha 1}\alpha + c_{D\alpha 2}\alpha^2 + c_{D\alpha 3}\alpha^3 + c_{D\delta e}\delta_e$$

$$c_C = c_{C\beta}\beta + c_{Cp}p^A + c_{Cr}r^A + c_{C\delta r}\delta_r$$

$$c_L = c_{Lo} + c_{L\alpha}\alpha + c_{Lq}q^A + c_{L\dot\alpha}\dot\alpha + c_{L\delta e}\delta_e$$

$$c_l = (c_{l\beta o} + c_{l\alpha\beta}\alpha)\beta + c_{lp}p^A + (c_{lr0} + c_{l\alpha r}\alpha)r^A$$

$$+ c_{l\dot\beta}\dot\beta + c_{l\delta r}\delta_r + c_{l\delta a}\delta_a$$

$$c_m = c_{mo} + c_{m\alpha}\alpha + c_{mq}q^A + c_{m\dot\alpha}\dot\alpha + c_{m\delta_e}\delta_e$$

$$c_n = c_{n\beta}\beta + (c_{npo} + c_{n\alpha p}\alpha)p^A + c_{nr}r^A$$

$$+ c_{n\dot\beta}\dot\beta + c_{n\delta r}\delta_r + c_{n\delta a}\delta a \tag{10}$$

Flight in the surface boundary layer is considered. The mean wind direction is constant and the windspeed magnitude can locally be approximated as having a linear variation with height. Choosing the Earth-fixed axis $O_E x_E$ in the vertical plane containing vector W_E^E, the wind field is defined by constant values of u_{Ez}^W, w_E^W and $\dot u_E^W = u_{Ez}^W \dot z_E$ and by the other components zero.

The scalar components of ω_B^A become

$$p^A = p - ½(L_{BV})_{12}u_{Ez}^W = p - u_{Ez}^W/2\cos\theta\sin\psi \tag{11a}$$

$$q^A = q - ½(L_{BV})_{22}u_{Ez}^W$$

$$= q - u_{Ez}^W/2(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) \tag{11b}$$

$$r^A = r - ½(L_{BV})_{32}u_{Ez}^W$$

$$= r - u_{Ez}^W/2(\cos\phi\sin\theta\sin\psi + \sin\phi\cos\psi) \tag{11c}$$

and the term $\dot u_E^W$ will be

$$\dot u_E^W = u_{Ez}^W \dot z_E = u_{Ez}^W\{w_E^W + V[\sin\beta\ \sin\phi$$

$$+ \sin\alpha\ \cos\beta\ \cos\phi)\cos\theta - \cos\alpha\ \cos\beta\ \sin\theta]\} \tag{12}$$

It should be noted that the influence of wind shear is represented by the dependence of aerodynamic forces on vector ω^A, Eqs. (7), (10), and (11), and by the existence of terms such as $(L_{WV})_{1l}\dot u_E^W$, due to windspeed variation along the aircraft path as well [Eqs. (7) and (12)].

Reference Steady-State Solution

A steady-state solution of Eq. (7) subject to the above assumptions is now determined. Since the parameters of this equation varies slowly with height z_E, and the motion is considered within a sufficiently thin layer of the atmosphere, the variation with height can be neglected and z_E considered a constant parameter. Therefore only the first nine elements of Eq. (7) will be considered. To define the steady-state solution of this equation, time derivatives of the variables are set equal to zero, and the steady states are then defined by a set of nine algebraic equations with the following 13 unknown constant quantities: T, V, β, α, p, q, r, ϕ, θ, ψ, δe, δr, and δa. Of these, V, β, ψ, and θ are selected as parameters to be added to z_E, u_{Ez}^W and w_E^W. Now the system of algebraic equations can readily be reduced to a nonlinear equation in α. Generally a single solution within the admissible domain of incidence values is of practical interest.

If the wind gradient is nonvanishing, the steady state is a constant translation relative to the air, superimposed on the variable speed of wind relative to the Earth. The resultant motion of the aircraft with respect to the Earth is curved and nonrotational viz. ϕ, θ and ψ are constant ($p = q = r = 0$).

The steady-state characteristics of a light airplane have been computed for $M = 0.2$, $\beta = 0$, $\dot z_E = -5$, 0 and 5 m/s and several directions of the wind ($\psi = 0$, 45, 90, 135, and 180 deg). Some of them are plotted in Fig. 1. The thrust required for the airplane in steady-state flight is seen to vary markedly with angle ψ; the amount and sense of variation depends on the vertical speed of the aircraft. In particular, the thrust is

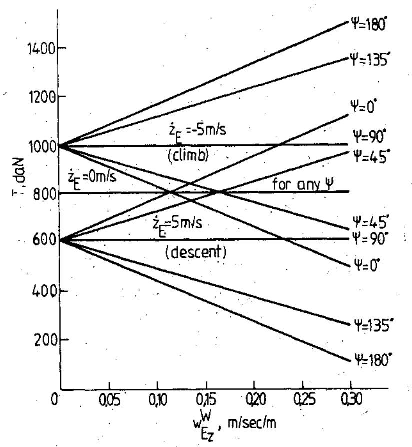

Fig. 1a Variation with vertical wind gradient of the steady-state values of thrust of the aircraft in landing configuration. $\psi = 0$ stands for tailwind and $\psi = 180°$ for headwind.

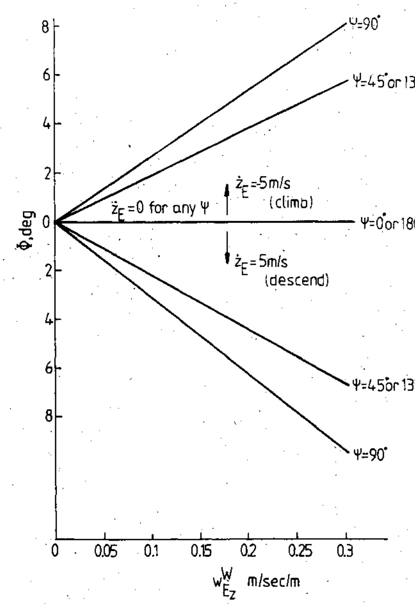

Fig. 1b Variation with vertical wind gradient of the steady-state values of bank angle of the aircraft in landing configuration. $\psi = 0$ stands for tailwind and $\psi = 180°$ for headwind.

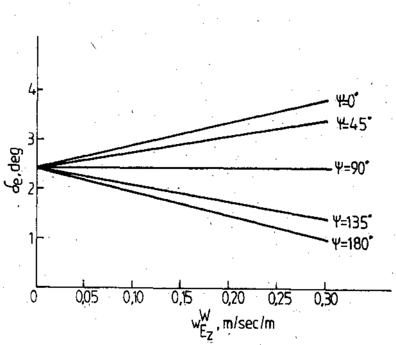

Fig. 1c Variation with vertical wind gradient of the steady-state values of elevator deflection of the aircraft in landing configuration. $\psi = 0$ stands for tailwind and $\psi = 180°$ for headwind.

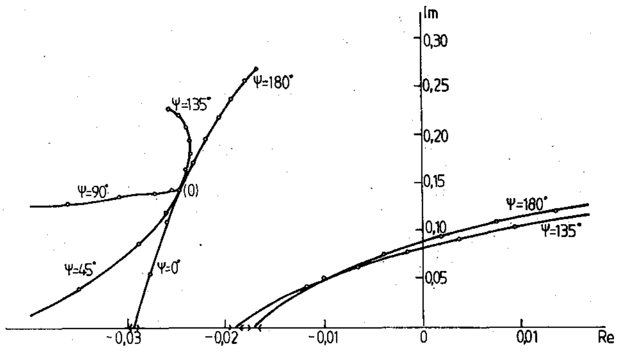

Fig. 2a Loci of the phugoid and spiral-ψ roots when the vertical wind gradient varies from $u_{Ez}^W = 0$ to 0.3 m/s/m by increments of 0.05 m/s/m for the aircraft in landing configuration, climb ($\dot{z}_E = -5$ m/s).

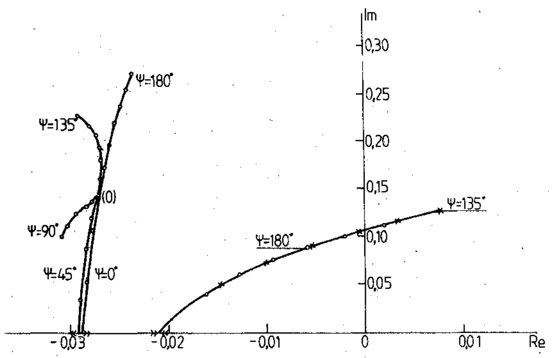

Fig. 2b Loci of the phugoid and spiral-ψ roots when the vertical wind gradient varies from $u_{Ez}^W = 0$ to 0.3 m/s/m by increments of 0.05 m/s/m for the aircraft in landing configuration, level flight ($\dot{z}_E = 0$).

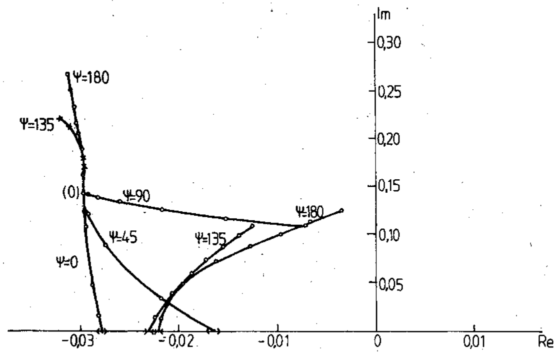

Fig. 2c Loci of the phugoid and spiral-ψ roots when the vertical wind gradient varies from $u_{Ez}^W = 0$ to 0.3 m/s/m by increments of 0.05 m/s/m for the aircraft in landing configuration, descent ($\dot{z}_E = 5$ m/s).

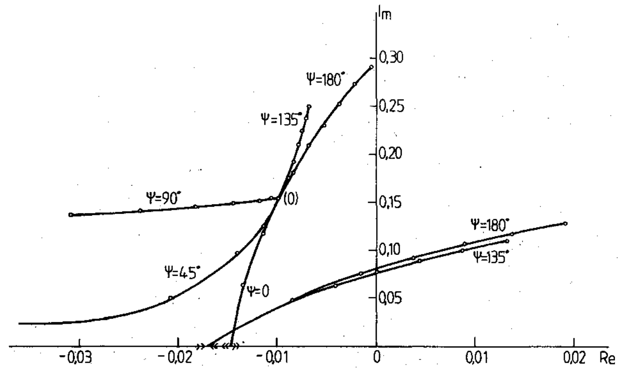

Fig. 3a Loci of the phugoid and spiral-ψ roots for the aircraft in takeoff configuration, climb ($\dot{z}_E = -5$ m/s).

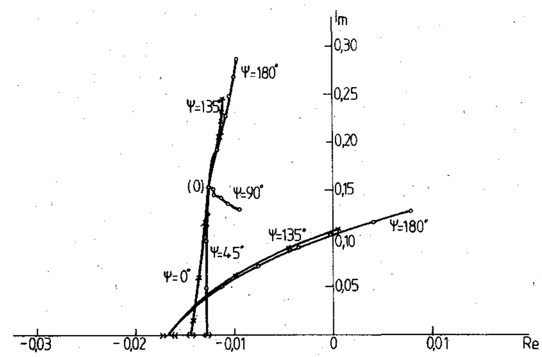

Fig. 3b Loci of the phugoid and spiral-ψ roots for the aircraft in takeoff configuration, level flight ($\dot{z}_E = 0$).

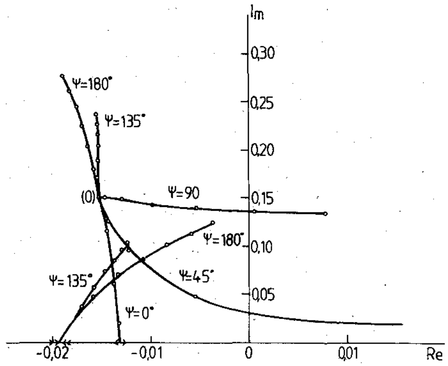

Fig. 3c Loci of the phugoid and spiral-ψ roots for the aircraft in takeoff configuration, decent ($\dot{z}_E = 5$ m/s).

independent of ψ in level flight. Maintaining a constant airspeed in a variable wind field accounts for this effect. In fact, the higher the vertical velocity of the aircraft, the greater the variation of the windspeed and the aircraft acceleration necessary to maintain a constant relative airspeed.

Flying Qualities

To study the flying qualities of an airplane in wind shear the linearized equations system about the steady-state solution is written, i.e.,

$$\dot{x} = Ax$$

where $x^T = |V, \beta, \alpha, p, q, r, \phi, \theta, \psi, \delta_e, \dot{\delta}_e, \delta_r, \dot{\delta}_r|$ and A is the matrix of the variational system. The z_E equation may also be added for level flight.

It is seen, from Eqs. (10) and (11), that aerodynamic derivatives with respect to the angular positions ϕ, θ, and ψ appear as a consequence of the nonisotropy of the wind field. Inertial terms, Eqs. (7) and (12), will also be included in A.

Now the eigenvalues and eigenvectors of matrix A are computed for each ψ and u_{Ez}^W considered. The values of u_{Ez}^W are taken in the interval from 0 to 0.3 m/s/m. The roll, the short period, and the Dutch roll modes, and those of the controls appeared to be practically unaffected by wind shear. It appeared, on the contrary, to have a heavy influence upon the phugoid and spiral modes, and the aperiodic mode allowing for the consideration of ψ as an additional variable of the system. This latter will subsequently be referred to as the ψ mode. As a matter of fact, in certain domains of the ψ values, the ψ mode is being coupled with the spiral one to generate an oscillatory mode, referred to hereunder as the spiral-ψ mode. For $\psi=0$ and $\psi=180$ deg, the situation is somewhat similar to the conventional one, inasmuch as the phugoid is mainly longitudinal and the spiral and ψ modes (or the spiral-ψ mode for $\psi=180$ deg) lateral-directional, as shown by an inspection of the corresponding eigenvectors. In the general case, however, when ψ differs both from zero and 180 deg, the eigenvectors prove to be rather inconclusive; it is hard, if, indeed, not altogether impossible to tell which is the phugoid and which the spiral-ψ mode, since apparently the longitudinal and the lateral-directional motion are coupled.

The root loci with the wind gradient u^W_{Ez} as a parameter are plotted in Figs. 2 and 3 for horizontal, descending and climbing flight and for several values of angle ψ between 0 and 180 deg. As is seen, the dynamic characteristics in wind shear improve in climb and deteriorate in descent as compared to the level flight. As for the effect of angle ψ, a headwind (such as $\psi=135$ or 180 deg) induces periodic phugoid and spiral-ψ modes whose frequencies increase with u^W_{Ez}, while a tailwind ($\psi=0$ or 45 deg), generally causes the eigenvalues to uncouple in four aperiodic modes; two are divergent, namely the ψ mode and one of the longitudinal slow modes. As already mentioned, these modes, however, are mainly lateral-directional or longitudinal, respectively, for only $\psi=0$ and 180 deg. Divergence was enhanced with the wind shear intensity.

A comparison of the results obtained for horizontal flight with $\psi=0$ and 180 deg has been made with those obtained by Etkin's method in Ref. 4 for the longitudinal motion. They proved to be coincident.

Conclusion

Wind shear is seen to account for coupling the longitudinal and the lateral-directional motions and for an additional mode as well. This new mode is obtained by the coupling of the spiral mode with that corresponding to the azimuth angle. The decisive influence of the wind direction upon the phugoid and spiral-ψ modes seems also to be a main characteristic of the flight in wind shear.

This paper is intended as an endeavor to treat coupled dynamics in wind shear and, therefore, no pretense of completeness is implied. Specifically, the contribution of the translation and the rotation of the atmospheric field is studied while rates of strain are neglected. It is the author's opinion that the consideration of these terms will only enhance the effects already mentioned.

Appendix
Transformation Matrices

$$L_{BV}=\begin{vmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi & \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ -\cos\phi\sin\psi & +\cos\phi\cos\psi & \\ \cos\phi\sin\theta\cos\psi & \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \\ +\sin\phi\sin\psi & -\sin\phi\cos\psi & \end{vmatrix}$$

$$L_{WB}=\begin{vmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{vmatrix}$$

$$L_{WV}=L_{WB}L_{BV}$$

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